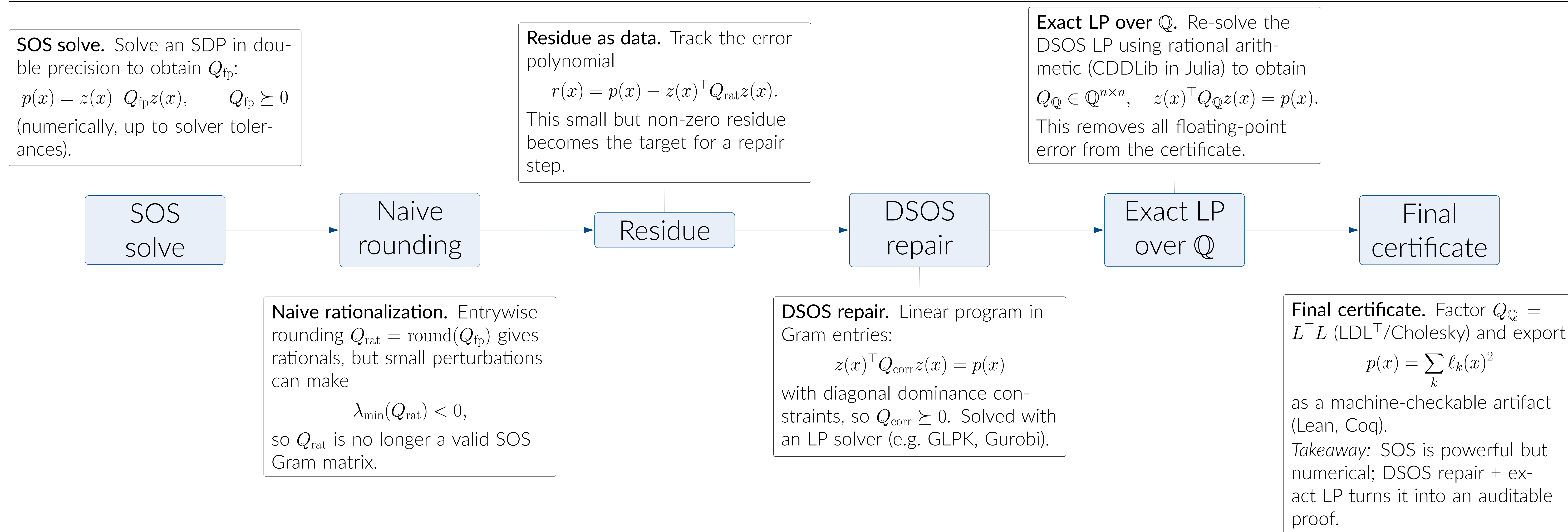


Barrier setting: what is the certificate about?

Positivstellensatz certificate template (CBF-style)	Failure modes (what breaks in practice)	SOS Encoding
<p>Let $K = \{x \mid g_i(x) \geq 0, i = 1, \dots, N\}$ be a basic semialgebraic set. For a polynomial barrier $B(x)$, we certify (schematically):</p> <p>(Interior) $B(x) \equiv \sum_i \alpha_i(x) p_i(x) + \alpha_0(x)$</p> <p>(Boundary) $-B(x) \equiv \sum_i \beta_i(x) q_i(x) + \beta_0(x)$</p> <p>(Invariance) $-\nabla B(x) \cdot f(x) + \lambda B(x) \equiv \sum_i \sigma_i(x) r_i(x) + \sigma_0(x)$</p> <p>where $\alpha_0, \beta_0, \sigma_0$ are SOS and p_i, q_i, r_i are products of the g_i's.</p> <p>$\alpha_i, \beta_i, \sigma_i, \dots \implies m(x)^\top Q_i m(x)$ Q_i should be positive semi-definite</p>	<p>Four common ways a “successful” SOS solution fails to become a proof:</p> <ol style="list-style-type: none"> 1. PSD non-rationalizability: Q is PSD but irrational; rounding breaks identity. 2. Slight indefiniteness: $\lambda_{\min}(Q) \approx 0$ (or slightly negative); formal proof needs strict PSD. 3. Identity drift after rounding: most common; residue $(x) \neq 0$ over $[x]$. 4. Tight margins: near-active constraints produce near-singular Grams; rounding amplifies drift. 	<p>We introduce repair slacks Σ_j constrained to DSOS/SDSOS:</p> $\text{LHS}_j(\widehat{B}) - \text{RHS}_j(\widehat{R}_j) - \Sigma_j(x) \equiv 0, \quad j \in \{1, 2, 3\}.$ <p>Concretely:</p> $B \equiv \sum \alpha_i p_i + \alpha_0 + \alpha_{\text{DSOS/SDSOS}},$ $-B \equiv \sum \beta_i q_i + \beta_0 + \beta_{\text{DSOS/SDSOS}},$ $-\nabla B \cdot f + \lambda B \equiv \sum \sigma_i r_i + \sigma_0 + \sigma_{\text{DSOS/SDSOS}}.$ $\alpha_i, \beta_i, \sigma_i, \dots \implies m(x)^\top Q_i m(x)$ <p>Q_i should be positive semi-definite</p>

Proof-producing SOS barrier pipeline



” From floating-point witnesses to exact, checkable proofs. ”